

Extra dimensions in LHC via mini-black holes: effective Kerr-Newman braneworld effects

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We solve Einstein equations on the brane to derive the exact form of the braneworld-corrected perturbations in Kerr-Newman singularities, using Randall-Sundrum and Arkani-Hamed-Dimopoulos-Dvali (ADD) models. It is a consequence of such models the possibility that Kerr-Newman mini-black holes can be produced in LHC. We use this approach to derive a normalized correction for the Schwarzschild Myers-Perry radius of a static $(4+n)$ -dimensional mini-black hole, using more realistic approaches arising from Kerr-Newman mini-black hole analysis. Besides, we prove that there are four Kerr-Newman black hole horizons in braneworld scenario we use, although only the outer horizon is relevant in the physical measurable processes. Parton cross sections in LHC and Hawking temperature are also investigated as functions of Planck mass (in the LHC range 1-10 TeV), mini-black hole mass and the number of large extra dimensions in braneworld large extra-dimensional scenarios. In this case a more realistic brane effect-corrected formalism can achieve more precisely the effective extra-dimensional Planck mass and the number of large extra dimensions — in Arkani-Hamed-Dimopoulos-Dvali model — or the size of the warped extra dimension — in Randall-Sundrum formalism.

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The hierarchy problem, concerning the large ratio of the weak scale mass and the Planck scale mass, can possibly shed new light on the existence of large extra dimensions and, in order to improve some theories that have not been tested yet, and their predictions — like compactification of extra dimensions of the order of the Planck length ($\approx 10^{-33}$ cm) — there has recently been proposed new approaches [1, 2, 3, 4, 5] suggesting that extra dimensions can be as large as around a millimeter. One of these approaches, Arkani-Hamed-Dimopoulos-Dvali (ADD) formalism, essentially suggests a higher-dimensional compact manifold to describe the universe, and explains how large extra dimensions could dilute the strength of gravity [4]. Although the gravitational force would be very strong in higher-dimensional spacetime, in the lower-dimensional effective theory gravity is weaker when the volume of the extra-dimensional compactified space is bigger. ADD predicted that this dilution of gravity into extra dimensions could possibly be so large that it can explain why the observed 4-dimensional gravity is so feeble. ADD models all have necessarily more than one extra curled up dimension – in order not to imply deviations in Newtonian gravity over

solar system distances — wherein a single brane accommodates Standard Model particles. ADD models propose that the fundamental mass scale determining gravity strength is close to a TeV, instead of the Planck scale mass. It represents the fundamental higher-dimensional Planck scale.

Another formalism concerning large extra dimensions is the Randall-Sundrum braneworld model [2, 3]. This formalism is effectively 5-dimensional, with one warped extra dimension, and it is based on a 5-dimensional reduction of $(1+9)$ -dimensional brane Horava-Witten theory [6, 7, 8, 9]. The idea that the universe is trapped on a membrane in some higher-dimensional spacetime may explain why gravity is so weak, and could be tested at high energy particle accelerators. In this context, the observable universe proposed by Randall and Sundrum, in one of their two models, can be described as a brane embedded in an AdS_5 bulk. At low energies gravity is localized on the brane and general relativity is recovered, but, at high energies, significant changes are introduced in gravitational dynamics, forcing general relativity to break down and to be overcome by a quantum gravity theory [10, 11]. A plausible reason for the gravitational force to appear to be so weak can be its dilution in possibly existing extra dimensions related to a bulk, where p -branes [12, 13, 14, 15, 16, 17] are embedded. p -branes are good candidates for braneworlds because they possess gauge symmetries [13, 14, 15] and automatically incorpo-

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rate a quantum theory of gravity.

In Randall-Sundrum formalism gravity is also diluted in warped extra-dimensional space, but this time due to graviton's probability function (GPF) that is dependent on the distance to the gravity brane of the theory. While in ADD models the GPF is equally distributed over all the extension of extra-dimensional space, and 4-dimensional gravity becomes diluted, in Randall-Sundrum formalism GPF varies due to the warped geometry. By the effective warp-factor in Randall-Sundrum formalism, all fundamental masses, expected to be of the order of M_P (the Planck scale mass) on an auxiliary (gravity) brane, become 10^{16} smaller, about a TeV magnitude. All masses are rescaled by the warp-factor, and the 5-dimensional Planck scale is 10^{16} TeV, whereas the M_P denotes now the effective warp-factor suppressed Planck scale.

In previous papers we have explored Randall-Sundrum formalism in the context of supermassive black holes (BHs) present in the nucleus of galaxies and quasars, and when they cause deviations from the 4D general relativity, these corrections should cause a small deviation in all BH properties [19, 20]. It has been already proposed the observation of braneworld BHs, and there are also other proposals on probing extra dimensions [21, 22]. Now we want to investigate these braneworld effects for the case of mini-BHs and their possible measurable consequences in Large Hadron Collider (LHC) observations [23]. It is claimed that if the fundamental Planck scale is a TeV scale, LHC will produce over 10^7 black holes per year [25].

Mini-BHs have radii that are much smaller than the size of extra dimensions, and therefore can be considered as totally embedded in a D -dimensional space. It is argued that as particles approach each other in LHC, their gravitational attraction increases steadily, and they can enter extra-dimensional space when they are extremely close. In that case it would allow gravity to increase and a mini-BH could form. The mass of a mini-BH can be no smaller than Planck mass, of order 22 micrograms, corresponding to mini-BH radius of the quantum fuzzy foam (10^{-33} m). Also, mini-BH lifetime can be about 10^{-26} s and its temperature, typically around 80 GeV $\approx 1.5 \times 10^{14}$ K, is much lower than it would be in a 4-dimensional spacetime, but still presents a high surface temperature though, at which mini-BHs would evaporate very rapidly into photons, electrons, and quarks, with energies ranging from 80 GeV down. It is believed that mini-BHs would either evaporate completely or leave a remnant, and lifetime estimates were obtained in [24]. Although the Standard Model does not put a constraint on the minimum size BH production by an accelerator, of about less than 10^{16} TeV, some astounding ideas suggest that gravity becomes stronger at small distances because of the effects of extra dimensions accessed only by gravity.

First of all the reconstruction of temperature as a function of mini-BH mass provides knowledge concerning the

extra dimensions of spacetime [25, 26, 27]. In the case of Planck scales close to TeV, the number of extra dimensions could thus be easily unraveled by the features of emitted particles. The mini-BHs production in LHC [28, 29] is an expected possibility and, as it is claimed, this will be the dominant effect in LHC under a quantum gravity theory [30, 31, 32]. Also, a complete characterization of the Hawking radiation in extra-dimensional scenarios, showing that graviton emission becomes relevant for a large number of extra dimensions, can be seen in [33, 34, 35, 36, 37]. Exact stationary axisymmetric solutions describing rotating BHs — with tidal charge as well as with electric charge — localized on the brane in RS braneworld model have been investigated in [38]. Also, new solutions of the Einstein-Maxwell equations that describes an electrically charged and slowly rotating BH in five dimensions was discussed in [39, 40, 41].

We consider both Randall-Sundrum and ADD mini-BH production, because they present different character, as pointed out by Stojkovic [37]. While ADD mini-BHs have the first phase Hawking radiation mostly in the bulk (and the second phase is mostly on the brane), existence of relaxation time during which the black hole loses the bulk components of angular momentum, and recoil effect to leave the brane, a Randall-Sundrum mini-BH have bulk radiation strongly suppressed, do not have any bulk components of angular momentum (absence of relaxation time) and cannot recoil and leave the brane.

The main aim of this paper is to show how to correct, considering braneworld effects in ADD and Randall-Sundrum models, the mini-BH Kerr-Newman radii (horizons), by solving Einstein equations on the brane. In the case we consider, when the radius of a BH on the brane is much smaller than the size scale of the extra dimensions, the BH can be well described by the classical solutions of higher-dimensional Einstein equations. Schwarzschild radius, in the context of Myers-Perry extra-dimensional formalism, is shown here to be significantly increased by tidal charge and spinning effects. The corrections we obtain give rise to more precise calculations concerning cross sections, Planck and mini-BHs masses, and Hawking temperature, contributing in this way to a more complete, precise and realistic analysis of mini-BH production in the next generation of particle colliders, such as LHC.

This article is organized as follows: in Section I Einstein field equations on the brane are presented together with the brane-corrected Newtonian potential. Reissner-Nordström BHs are also considered in a braneworld scenario viewpoint. In Section II by inputting a radial coordinate-valued function to perturb a Kerr-Newman singularity, we solve Einstein field equations on the brane to find the explicit form of these perturbations. Various graphics are depicted to illustrate the corrections in Kerr-Newman radii as functions of the BH angular momentum, both in Randall-Sundrum and ADD braneworld scenarios, for various fixed values of the BH charge. Also, cross sections are now corrected by braneworld effects and it

gives rise to more realistic analysis of LHC collisions products, such as mini-BH and particle production via mini-BH decay. The corrections in Kerr-Newman mini-BH radii are analyzed separately in a Randall-Sundrum braneworld scenario — where there is $n = 1$ warped extra dimension only — and in ADD scenario with $n = 2, 4$ and 6 extra dimensions, as functions of Planck masses in the range 1-10 TeV, as well as BH masses and the BH spinning parameter. Graphics of parton cross sections are also shown as functions of Planck masses in the range 1-10 TeV and BH masses. Finally in Section III Kerr-Newman mini-BH Hawking evaporation and temperature are investigated in a braneworld scenario. All graphics present our results in explicit comparison with Schwarzschild BH described by Myers-Perry standard results — see, e.g., [25, 26, 27].

I. MINI-BLACK HOLES ON THE BRANE

In a braneworld scenario the Einstein field equations read [19, 20, 42, 43]

$$G_{\mu\nu} = -\frac{1}{2}\Lambda_5 g_{\mu\nu} + \frac{1}{4}\kappa_5^4 \left[TT_{\mu\nu} - T_\nu^\alpha T_{\mu\alpha} + \frac{1}{2}g_{\mu\nu}(T^2 - T_{\alpha\beta}^{\alpha\beta}) \right] - E_{\mu\nu},$$

where $T = T_\alpha^\alpha$ denotes the trace of the energy-momentum tensor, Λ_5 denotes the 5-dimensional cosmological bulk constant, and $E_{\mu\nu}$ denotes the ‘electric’ components of the Weyl tensor, which can be expressed by means of the extrinsic curvature components $K_{\mu\nu} = -\frac{1}{2}\mathcal{L}_n g_{\mu\nu}$ by [44]

$$E_{\mu\nu} = \mathcal{L}_n K_{\mu\nu} + K_\mu^\alpha K_{\alpha\nu} - \frac{1}{\ell^2} g_{\mu\nu}, \quad (1)$$

where ℓ denotes the bulk curvature radius. It corresponds equivalently to the effective size of the extra dimension probed by a 5D graviton [2, 3, 43]. Let κ_5 be defined as $\kappa_5 = 8\pi G_5$, where G_5 denotes the 5-dimensional Newton gravitational constant, which can be related to the 4-dimensional gravitational constant G by $G_5 = G\ell_{\text{Planck}}$, where $\ell_{\text{Planck}} = \sqrt{G\hbar/c^3}$ is the Planck length.

As indicated in [3, 43], “... table-top tests of Newton’s law currently find no deviations down to the order of 0.1 mm...”, so that $\ell \lesssim 0.1$ mm [31, 45] provides a more accurate magnitude limit improvement on the bulk curvature ℓ . The Weyl ‘electric’ term $E_{\mu\nu}$ carries an imprint of high-energy effects sourcing KK modes. The component $E_{\mu\nu}$ also carries information about the collapse process of BHs. In the perturbative analysis of Randall-Sundrum positive tension 3-brane, KK modes consist of a continuous spectrum without any gap [18]. It generates a correction in the gravitational potential $V(r) = \frac{GM}{c^2 r}$ to 4D gravity at low energies by extra-dimensional effects [43],

which is given by [2, 3], for $r \gg \ell$

$$V(r) = \frac{GM}{c^2 r} \left[1 + \frac{2\ell^2}{3r^2} + \mathcal{O}\left(\frac{\ell}{r}\right)^4 \right], \quad (2)$$

and for $r \ll \ell$,

$$V(r) \approx \frac{GM\ell}{c^2 r^2}. \quad (3)$$

The KK modes that generate these corrections are responsible for a nonzero $E_{\mu\nu}$. This term carries the modification to the weak-field field equations, as we have already seen.

For a static spherical metric on the brane given by

$$g_{\mu\nu} dx^\mu dx^\nu = -F(r) dt^2 + \frac{dr^2}{H(r)} + r^2 d\Omega^2, \quad (4)$$

the projected electric component Weyl term on the brane is given by the expressions [19, 20, 43]

$$\begin{aligned} E_{00} &= \frac{F}{r} \left(H' - \frac{1-H}{r} \right), \quad E_{rr} = -\frac{1}{rH} \left(\frac{F'}{F} - \frac{1-H}{r} \right), \\ E_{\theta\theta} &= -1 + H + \frac{r}{2} H \left(\frac{F'}{F} + \frac{H'}{H} \right). \end{aligned} \quad (5)$$

Note that in Eq.(4) the Schwarzschild metric is recovered if $F(r)$ equals $H(r)$. It is well known that the most general solutions of Eq.(4), in the case of a Reissner-Nordström (RN) BH, are given by [43, 46]

$$F(r) = 1 - \frac{2GM}{c^2 r} - \psi(r) = 1 - \frac{2GM}{c^2 r} + \frac{2G\ell Q^*}{c^2 r^2}, \quad (6)$$

where M and Q^* denote respectively the mass and the tidal charge induced by the bulk of an effective RN BH. The bulk induces a tidal charge BH on the brane. The radial coordinate-valued $\psi(r)$ is obtained when one substitutes $F(r)$ in Einstein equations on the brane (see Eq.(13)). We use in the next Section an analogous procedure to obtain the braneworld form of Kerr-Newman metric.

As asserted in [43], a negative Q^* strengthens the gravitational field, since it arises from the source mass M on the brane. By contrast, in the RN solution of general relativity, $Q^* \sim q^2$, where q denotes the RN mini-BH electric and tidal charges, and this weakens the gravitational field. Negative tidal charge also preserves the spacelike nature of the singularity. The tidal charge BH metric does not satisfy the far-field r^{-3} correction to the gravitational potential, however Eq.(6) shows the correct 5D behavior of the potential at short distances, so that the tidal-charge metric could be a good approximation in the strong-field regime for small BHs [43], as seen in Eq.(3). Eqs. (3) and (6) also imply, for extremal BHs defined by the relation

$$Q^* = -2M, \quad (7)$$

that RN BHs in braneworld models also have two — inner and outer — horizons, denoted by R_{RNbrane}^\pm , which can be obtained by fixing $F(r) = 0$ in Eq.(6). It results in [43]

$$R_{\text{RNbrane}}^\pm = \frac{GM}{c^2} \pm \frac{1}{c} \left[\frac{G^2 M^2}{c^2} - 2\ell G Q^* \right]^{1/2}. \quad (8)$$

There are two horizons given by Eq.(8) and the charge Q^* is constrained by the real-valued square-root in the same equation. In the next Section we will see that, in a braneworld scenario Kerr-Newman BHs presents four horizons.

II. REALISTIC KERR-NEWMAN MINI-BHS AT LHC

Our assumption is now to consider a realistic mini-BH to be produced in LHC, which is unlikely static, and the realistic conception is to proceed by considering a spinning, charged Kerr-Newman mini-BH. It has been demonstrated [47] that half a mini-BH mass is emitted when it is highly rotating, confirming it is of primordial importance to take into account the angular momentum of BHs. The Kerr-Newman metric, in Boyer-Lindquist coordinates, describing the neighborhood of a spherical rotating BH with mass M , angular momentum J and charge Q , is given by

$$g_{\mu\nu}^{\text{K-N}} = \begin{pmatrix} \gamma/\rho^2 & 0 & 0 & -\omega\beta^2 + \alpha \\ 0 & \rho^2/\Delta & 0 & 0 \\ 0 & 0 & \rho^2 & 0 \\ -\omega\beta^2 + \alpha & 0 & 0 & \beta^2 \end{pmatrix} \quad (9)$$

where

$$\begin{aligned} \Delta &= r^2 + \frac{a^2}{c^2} - 2\frac{GM}{c^2}r + Q^2, \quad \rho^2 = r^2 + \frac{a^2}{c^2}\cos^2\theta, \\ \Sigma^2 &= \left(r^2 + \frac{a^2}{c^2}\right)^2 - \frac{a^2}{c^2}\Delta\sin^2\theta, \quad \beta = \frac{\Sigma}{\rho}\sin\theta, \\ \omega &= \frac{2aGMr}{c^3\Sigma^2}, \quad \alpha = \frac{a\sin^2\theta Q^2}{c\rho^2}, \\ \gamma &= \frac{a^2\sin^2\theta}{c^2} - \Delta. \end{aligned} \quad (10)$$

The spinning parameter a is defined as $a = \frac{J}{Mc}$. In order to write the Kerr-Newman metric in a diagonal form, when we solve the characteristic eigenvalue equation associated with Eq.(9), the eigenvalues are given by

$$\begin{aligned} \lambda_2 &= \frac{\rho^2}{\Delta}, \quad \lambda_3 = \rho^2, \\ \lambda_{1,4} &= \frac{1}{2} \left[\gamma/\rho^2 + \beta^2 \right. \\ &\quad \left. \pm \left(\gamma^2/\rho^4 + 4(\alpha - \omega\beta^2) - 2\beta^2\gamma/\rho^2 + \beta^4 \right)^{1/2} \right]. \end{aligned}$$

Now we must impose a condition — arising when the eigenvalue characteristic equation is solved — for *real* eigenvalues, i.e.,

$$\gamma^2/\rho^4 + 4(\alpha - \omega\beta^2) - 2\beta^2\gamma/\rho^2 + \beta^4 \geq 0, \quad (11)$$

from which the Kerr-Newman metric is given in a diagonal form:

$$\begin{aligned} g^{\text{K-N}} &= g_{\mu\nu}^{\text{K-N}} dx^\mu dx^\nu \\ &= \lambda_1 dt'^2 + \frac{dr^2}{\Delta/\rho^2} + \rho^2 d\theta^2 + \lambda^4 d\phi'^2. \end{aligned} \quad (12)$$

Here $d\phi'$ and dt' are 1-form fields on the 3-brane respectively related to $d\phi$ and dt by the new eigenvectors in the associated directions defined by the eigenvalue equation associated with Eq.(9).

In order to obtain the correction of the Kerr-Newman radii, by braneworld effects, we follow the idea presented in Eq.(6). Besides, it is well known that a particular way to express the vacuum field equations on the brane is [43]

$$E_{\mu\nu} = -R_{\mu\nu}, \quad R_\mu{}^\mu = 0 = E_\mu{}^\mu, \quad \nabla^\nu E_{\mu\nu} = 0. \quad (13)$$

Defining $\xi(r)$ as the deviation from a Kerr-Newman form Δ/ρ^2 — the term in the denominator of dr^2 in Eq.(12) — we can now *solve* Eqs. (13), via Eqs.(5), for the particular case where $\mu = \nu = 0$, obtaining

$$\xi(r) = \frac{2G\ell Q^*}{c^2 r(r + a/c)} \quad (14)$$

as one solution, in such a way that in a static limit ($a \rightarrow 0$) the correction in the gravitational potential satisfies Eq.(3). Imposing that the charge in these mini-BHs are induced *a priori* by bulk effects, we can define $Q = GQ^*/c^2$. Now, the corrections $R_{\text{K-Nbrane}}$ in the Kerr-Newman radii are obtained via the deviated Kerr-Newman form, as

$$\frac{\Delta}{\rho^2} + \xi(R_{\text{K-Nbrane}}) = 0. \quad (15)$$

By expanding the expression above and using Eqs.(10) and (14) we obtain

$$\begin{aligned} R_{\text{K-Nbrane}}^4 &+ \Gamma_3 R_{\text{K-Nbrane}}^3 + \Gamma_2 R_{\text{K-Nbrane}}^2 \\ &+ \Gamma_1 R_{\text{K-Nbrane}} + \Gamma_0 = 0, \end{aligned} \quad (16)$$

where

$$\begin{aligned} \Gamma_3 &= a/c - 2GM/c^2, \\ \Gamma_2 &= a^2/c^2 + 2\ell Q - 2aGM/c^3 + Q^2, \\ \Gamma_1 &= a^3/c^3 + aQ^2/c, \\ \Gamma_0 &= 2G\ell Q^* a^2 \cos^2\theta/c^2. \end{aligned} \quad (17)$$

A mini-BH not corrected by extra-dimensional tidal charge effects can be simply described by a Kerr mini-BH: the horizon radius comes from the expression obtained by making $Q = 0$ in Eq.(12). Using natural

units ($c = 1$, $G = 1$), by solving Eq.(16) we find results for the normalized brane-corrected Kerr-Newman radius $K(M, a, Q)$. In what follows, this normalized horizon will be used to make corrections to a conventional Schwarzschild radius R_S , in order to find a Kerr-Newman BH by $R_{K-N\text{brane}} = \frac{1}{2}K(M, a, Q)R_S$. Eq.(16) gives four solutions for $R_{K-N\text{brane}}$ and we depict below three graphics respectively representing the inner $R_{K-N\text{brane}}$ horizon corrections, intermediary $R_{K-N\text{brane}}$ radius corrections, and the outer $R_{K-N\text{brane}}$ horizon corrections, as functions of the spinning parameter a , for specific values of the charge Q^* . Solutions for the normalized horizon $K(M, a, Q)$ with respect to the spinning parameter factor a is presented in Figures (1), (2), and (3), respectively for inner, intermediate, and outer Kerr-Newman horizons.

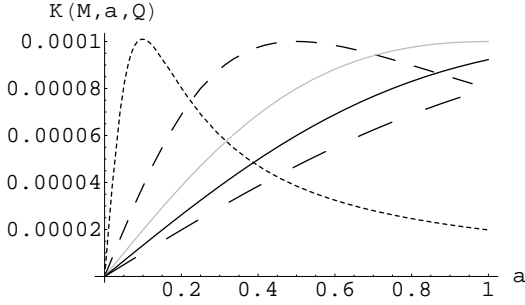


FIG. 1: Graphic of the brane effect-corrected normalized inner horizon $K(M, a, Q) \times a$ for different values of Q^* . For the very short dashed line: $Q^* = -0.1$; for the short dashed line: $Q^* = -0.5$; for the full gray $Q^* = -1$; for the full black line: $Q^* = -1.5$; and for the long dashed line: $Q^* = -2$.

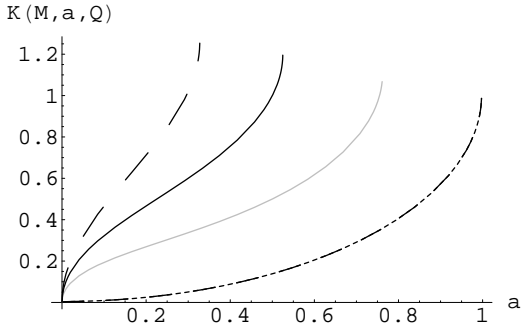


FIG. 2: Graphic of the brane effect-corrected normalized horizon $K(M, a, Q) \times a$ for different values of Q^* . These are solutions for a intermediate horizon. For the very short dashed line: $Q^* = -0.1$; for the short dashed line: $Q^* = -0.5$; for the full gray $Q^* = -1$; for the full black line: $Q^* = -1.5$; and for the long dashed line: $Q^* = -2$. In this case, the spin of the mini-BH is constrained by the electric and tidal charge. The maximum a obtained for each graphic is: $a = 0.32806$ for $Q^* = -2$, $a = 0.52572$ for $Q^* = -1.5$, $a = 0.76550$ for $Q^* = -1$, $a = 0.93842$ for $Q^* = -0.5$ and $a = 0.99753$ for $Q^* = -0.1$ (which practically coincides with the $Q^* = -0.5$ case). In these respective values for a , the derivative of $K(M, a, Q)$ related to a tends to infinity.

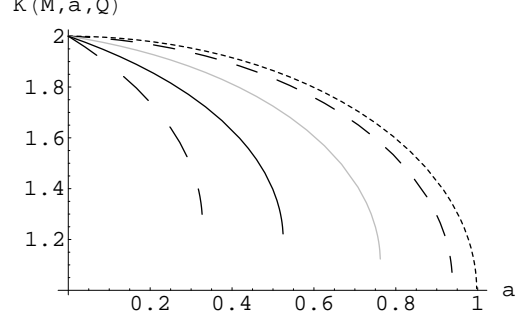


FIG. 3: Graphic of the brane effect-corrected Kerr-Newman outer horizon in the form $c^2 R_{K-N} / GM \times a$ for different values of Q^* . For the very short dashed line: $Q^* = -0.1$; for the short dashed line: $Q^* = -0.5$; for the full gray $Q^* = -1$; for the full black line: $Q^* = -1.5$; and for the long dashed line: $Q^* = -2$. In this case, the spin of the BH is constrained by the electric and tidal charge. The maximum a obtained for each graphic is: $a = 0.32791$ for $Q^* = -2$, $a = 0.52766$ for $Q^* = -1.5$, $a = 0.76585$ for $Q^* = -1$, $a = 0.95826$ for $Q^* = -0.5$ and $a = 0.99866$ for $Q^* = -0.1$.

These corrections can be applied to the Schwarzschild radius R_S of a Myers-Perry $(4+n)$ -dimensional BH [25, 30, 48], resulting in

$$R_{K-N\text{brane}} = \frac{1}{2}K(M, a, Q)R_S$$

$$= \frac{K(M, a, Q)}{2\sqrt{\pi}M_{P(4+n)}} \left[\frac{M_{BH}}{M_{P(4+n)}} \left(\frac{8\Gamma\left(\frac{n+3}{2}\right)}{n+2} \right) \right]^{\frac{1}{n+1}} \quad (18)$$

where $\Gamma(m)$ denotes the usual Euler Gamma function, $M_{P(4+n)}$ is the $(4+n)$ -dimensional Planck mass and M_{BH} is the mini-BH mass. Both $M_{P(4+n)}$ and M_{BH} are given in TeV. Expression (18) is normalized ($c = 1$, $\hbar = 1$) and to obtain the value of $R_{K-N\text{brane}}$ in meters it is necessary to multiply the expression by $\hbar c = 1.973 \times 10^{-13}$ MeV m. The three solutions for $K(M, a, Q)$, represented by the graphics of Figs. (1), (2), and (3) generate three horizons, which could be related to quantum BH theories [49]. Only the outer horizon expressed by Fig. (3) contributes to the total cross section (see Fig. (4)).

Denoting by M_* the strength of higher-dimensional gravity, $M_P \approx 10^{16}$ TeV sets the strength of 4-dimensional gravity, and V^n the volume of the higher-dimensional space, the 3-space-dimensional Newton's force law is recovered by the identification [25, 26, 27, 43]

$$M_P^2 = M_*^{n+2} V^n.$$

Important differences concerning the number of extra dimensions in ADD and Randall-Sundrum formalism resides in the last expression. In the one hand, if there were only one large extra dimension in ADD model, gravity measured at, e.g., the solar system, would have a 5-dimensional non-observed behavior. With $n = 2$ extra dimensions the size of extra dimensions would be a

tenth of a millimeter. On the other hand, the Randall-Sundrum model is effectively 5-dimensional, with one extra warped dimension. In the graphics below we present our results for certain number of extra dimensions, and it is implicit that, when we consider $n = 1$ extra dimension it is associated with Randall-Sundrum models, while for $n = 2, \dots, 6$ extra dimensions our results represent ADD models.

Considering two partons with the center of mass energy $\sqrt{s} = M$ moving in opposite directions, semi-classical reasoning suggests that the impact parameter is less than the Schwarzschild radius — in the case of an extremal Kerr-Newman mini-BH — or less than the outer Kerr-Newman horizon — when the Kerr-Newman mini-BH is *not* extremal — a mini-BH with the mass M arises. For the Schwarzschild BH particular case, see [25, 26, 27]. Parton distribution functions at LHC gives the total cross section σ for production of BHs with $M_{BH} > M_P$ in the range $1 \text{ pb} < \sigma < 15 \text{ pb}$, for $1 \text{ TeV} < M_P < 5 \text{ TeV}$ and it varies around 10% for n — the number of extra dimensions in ADD models — between 2 and 7 [26, 27].

By elementary geometrical arguments, the total cross section is given by [31, 48]

$$\sigma(M_{BH}) \approx \pi R_{K-N}^2 \approx \frac{K^2(M, a, Q)}{4M_{P(4+n)}^2} \left[\frac{M_{BH}}{M_{P(4+n)}} \left(\frac{8\Gamma\left(\frac{n+3}{2}\right)}{n+2} \right) \right]^{\frac{2}{n+1}}.$$

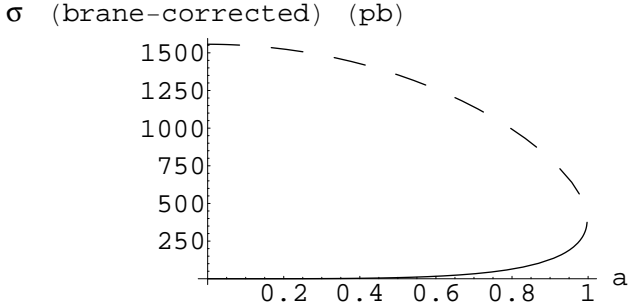


FIG. 4: Brane effect-corrected Kerr-Newman cross section σ for the inner horizon (full line) and for the outer horizon (dashed line) for various angular momenta a . Significant contributions by the inner horizon just happen when $a \rightarrow 1$. Thus, the effective contribution for the total cross section is only given by the outer horizon. This graphic presents an example for $n = 3$ extra-dimensional ADD model, $M_P = 1 \text{ TeV}$, $M_{BH} = 5 \text{ TeV}$, $Q^* = -0.1M_{BH}$.

Now, in Figs. (5, 6, 7, 8, 9) we illustrate the brane effects in Kerr-Newman mini-BHs — for different values of charge — as functions of mini-BH mass, Planck mass and spinning parameter (respectively in Randall-Sundrum and $n = 2, 4$ and 6 extra-dimensional ADD models). *In these graphics the wide gray line indicates the*

standard Myers-Perry approach for Schwarzschild mini-BHs. We show in this way a more realistic and precise prediction concerning mini-BH production in LHC, since our approach does not neglect spin and charge effects.

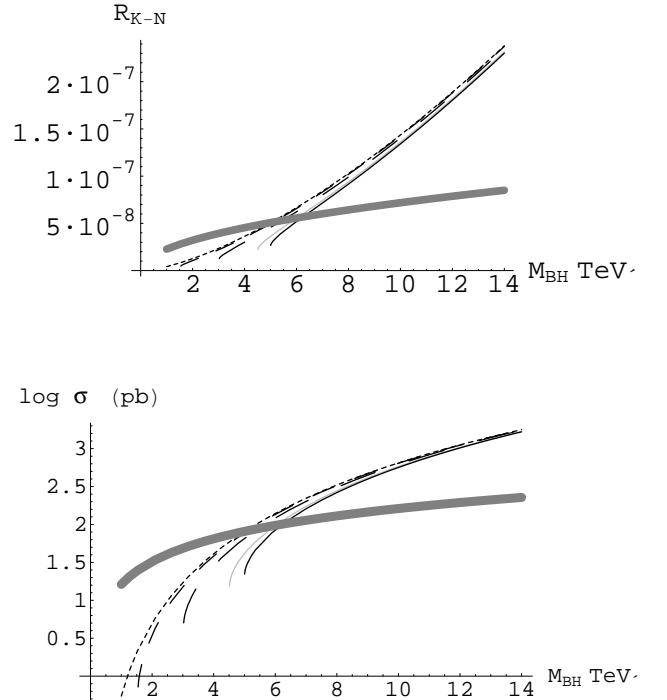


FIG. 5: Graphics of the brane effect-corrected Kerr-Newman horizon R_{K-N} of the produced mini-BH and its associated cross section, for various momenta a versus BH masses, and its associated parton cross section for Randall-Sundrum braneworld model, where there is one warped extra dimension. Here we present an example for $M_P = 4 \text{ TeV}$, and $Q^* = -0.1M_{BH}$, with short dashed line for $a = 0.1$, dashed line for $a = 0.3$, long dashed line for $a = 0.6$, gray full line for $a = 0.9$, full black line for $a = 0.997$.

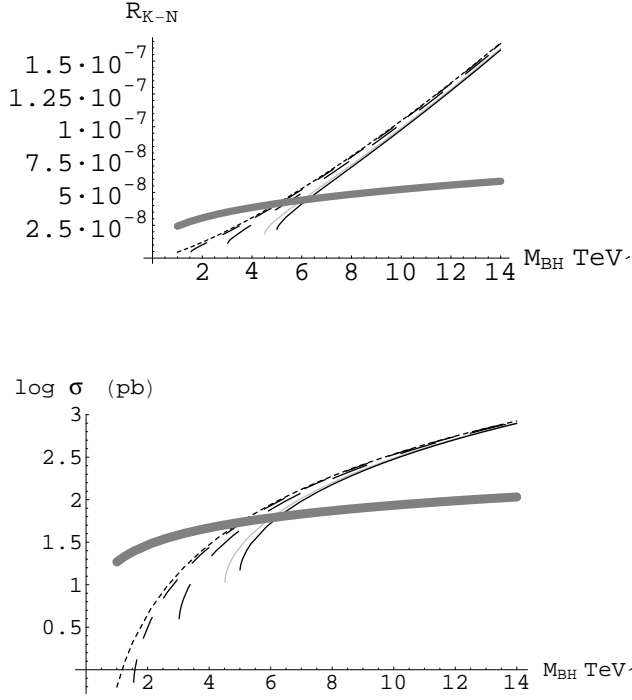


FIG. 6: Graphics of the brane effect-corrected Kerr-Newman radius and its associated parton cross section for $n = 2$ ADD model for some BH masses, with $M_P = 1$ TeV, and for various angular momenta: short dashed line for $a = 0.1$, dashed line for $a = 0.3$, long dashed line for $a = 0.6$, gray full line for $a = 0.9$, full black line for $a = 0.997$.

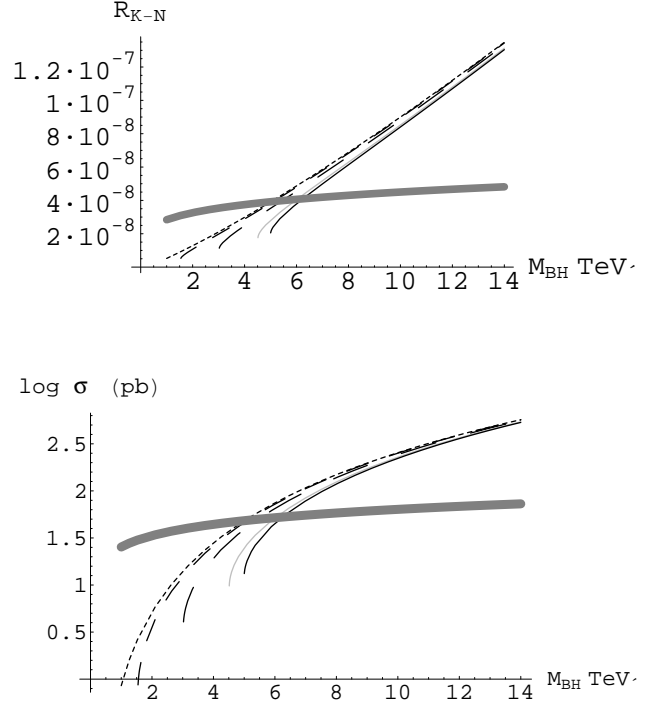


FIG. 7: Graphics of the brane effect-corrected Kerr-Newman outer horizon and its associated parton cross section in $n = 4$ ADD model for some mini-BH masses, with $M_P = 1$ TeV, and for various angular momenta: short dashed line for $a = 0.1$, dashed line for $a = 0.3$, long dashed line for $a = 0.6$, gray full line for $a = 0.9$, full black line for $a = 0.997$.

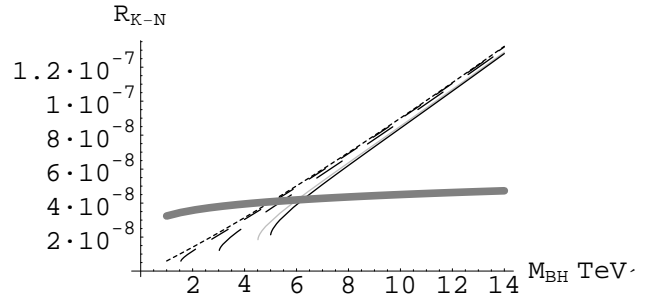


FIG. 8: Graphic of the brane effect-corrected Kerr-Newman outer horizon for extra-dimensional $n = 6$ ADD model for some BH masses, with $M_P = 1$ TeV, and for various angular momenta: short dashed line for $a = 0.1$, dashed line for $a = 0.3$, long dashed line for $a = 0.6$, gray full line for $a = 0.9$, full black line for $a = 0.997$.

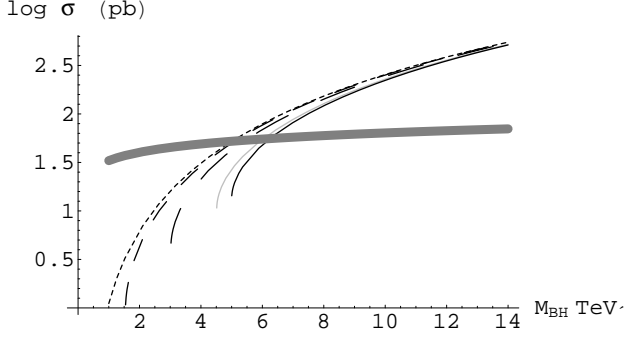


FIG. 9: Cross sections for $n = 6$ extra dimensions in ADD model, short dashed line for $a = 0.1$, dashed line for $a = 0.3$, long dashed line for $a = 0.6$, gray full line for $a = 0.9$, full black line for $a = 0.997$.

In Fig. (10) we show the parton cross section for the case of Schwarzschild mini-BH Myers-Perry production within LHC range of masses, and in Fig. (11) we show the parton cross section for the case of production of a spinning charged mini-BH within LHC range of masses in braneworld Randall-Sundrum scenario. We present the same graphics for $n = 2$ and $n = 6$ extra dimensions, respectively in Fig. (12), and in these cases, we obtain an attenuation in the cross section values when the BH mass increases.

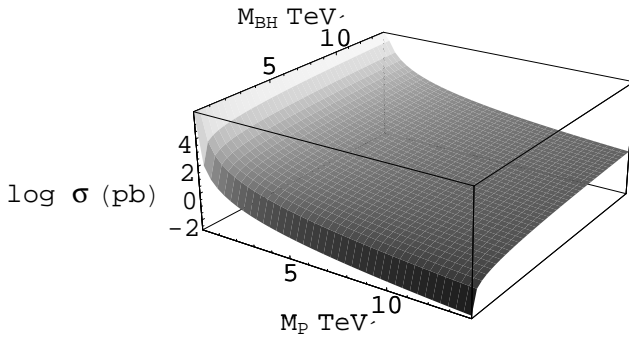


FIG. 10: Graphic of parton cross section in picobarn for various values of Planck mass M_P and BH mass M_{BH} for the Schwarzschild Myers-Perry model. The mini-BH formed by the scattering has $a = 0.5$ and is charged ($Q^* = -1M_{BH}$), for the Randall-Sundrum braneworld model.

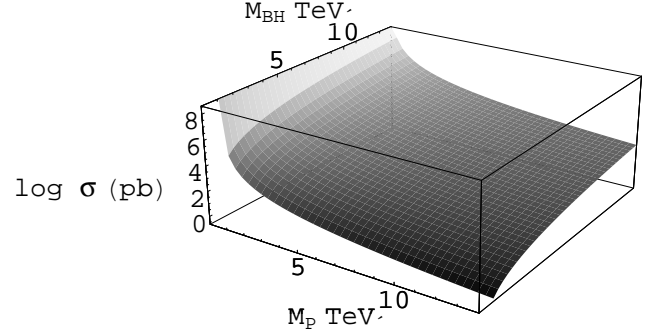


FIG. 11: Graphic of parton cross section in picobarn for various values of Planck mass M_P and BH mass M_{BH} . The mini-BH formed by the scattering has $a = 0.5$ and is charged with $Q^* = -1M_{BH}$, for the Randall-Sundrum braneworld model.

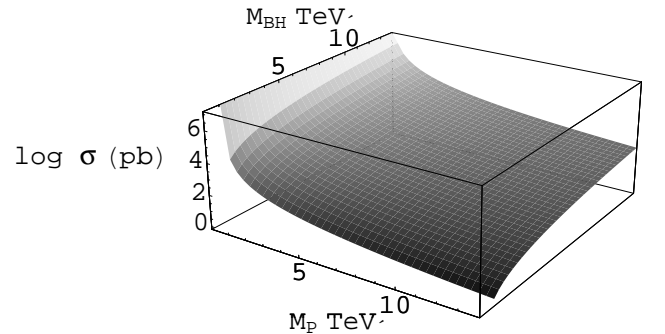
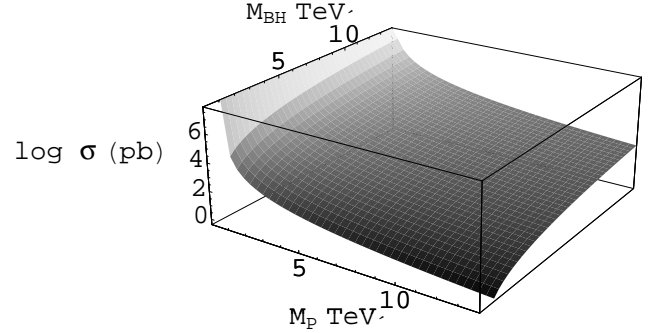


FIG. 12: Graphics of parton cross section in picobarn for various values of Planck mass M_P and BH mass M_{BH} . The mini-BH formed by the scattering has $a = 0.5$ and is charged with $Q^* = -1M_{BH}$, in respectively $n = 2$ and $n = 6$ extra dimensions in ADD model.

In Fig. (13) we show the parton cross section for the case of production of a spinning mini-BH within the LHC range of masses, and for the case where the scenario is

a Randall-Sundrum braneworld model; here we show explicitly the effect of the spinning parameter a . We present the same graphic for higher dimensions in Figs. (14) and (15). It is notorious the interesting effect caused by the spinning parameter a in the cross section. The higher the number of extra dimensions, for instance $n = 6$ as illustrated in Fig.(15), the faster the bending of the graphic is, as the spinning parameter $a \rightarrow 1$.

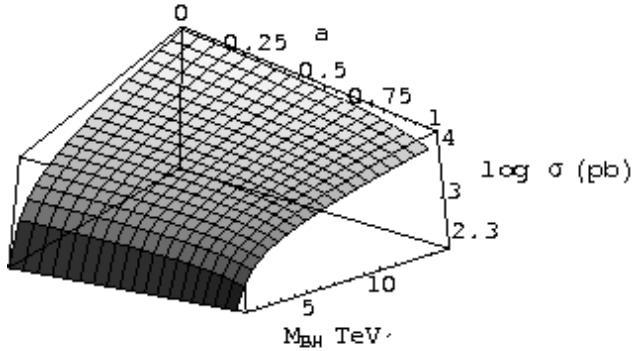


FIG. 13: Graphic of parton cross section in picobarn for various values of spinning parameter a and BH mass M_{BH} . The mini-BH formed by the scattering is charged with $Q^* = -1M_{BH}$. In this case we consider the Randall-Sundrum braneworld model.

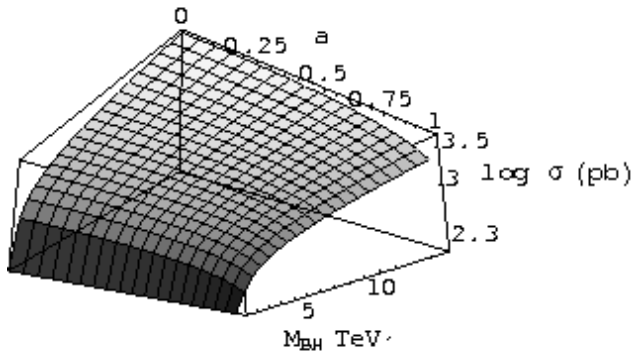


FIG. 14: Graphic of parton cross section in picobarn for various values of a and the BH mass M_{BH} . Note the interesting effect caused by the spinning parameter a in the cross section. Here we consider $n = 2$ extra-dimensional ADD model, the mini-BH is electrically and tidally charged by the bulk with $Q^* = -0.1M_{BH}$.

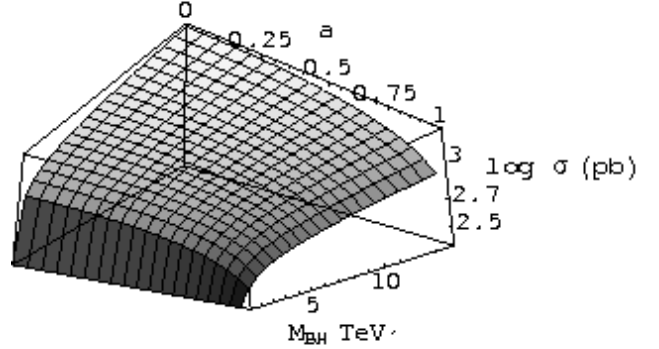


FIG. 15: Graphic of parton cross section in picobarn for various values of a and the BH mass M_{BH} . Note the interesting effect caused by the spinning parameter a in the cross section. In this case $n = 6$ extra-dimensional ADD model the bending is more intense. The mini-BH has $Q^* = -0.1M_{BH}$.

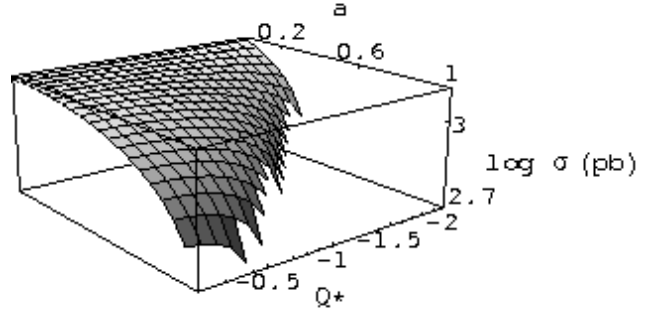


FIG. 16: Parton cross sections in picobarn for various values of Q^* and a , explicitly showing the influence in the scattering if one considers a realistic Kerr-Newman mini-BH. The charge constrains the spin.

III. BH EVAPORATION AND HAWKING RADIATION IN KERR-NEWMAN MINI-BHS

BHs can evaporate by emitting pairs of virtual photons at the event horizon, with one of the photon escaping the BH gravity [50], and have black-body spectrum with the Hawking temperature $\frac{\hbar c}{4\pi k R_S}$. Hawking evaporation is significant only in the case of small BHs, where the tidal effect becomes so intense near the surface that the particle pairs produced by quantum vacuum fluctuations may be broken, one particle falling into the BH and the other being projected outwards.

BH production in LHC is expected immediately above the M_P threshold, since at energies below M_P other quantum effects overcome BH production. In [26, 27] it was shown how the relationship between M_{BH} and Hawking temperature can unravel Planck mass and the dimensionality n of extra dimensions, independently of

the geometry of extra dimensions. Although when a BH event horizon is observed by its Hawking radiation it looks like a fuzzy sphere (see [51]) — and in the classical limit the event horizon looks locally like a non-commutative plane with non-commutative parameter dictated by the Planck length — in particular at M_P energies BHs are quantum objects and here we use as in [27] semi-classical arguments. Besides, mini-BHs produced at LHC would be light and extremely hot, with a Hawking temperature $T_H \approx 100$ GeV, and evaporate almost instantaneously, mainly via Hawking radiation.

Evaporation of a BH in n large extra dimensions occurs at Hawking temperature T_H , given in our model by

$$T_H = \frac{2M_P}{K(M, a, Q)} \left(\frac{M_{BH}}{M_P} \frac{n+2}{8\Gamma(\frac{n+3}{2})} \right)^{\frac{1}{n+1}} \frac{n+1}{4\sqrt{\pi}}. \quad (19)$$

We depict below T_H for different values of Kerr-Newman mini-BH charges:

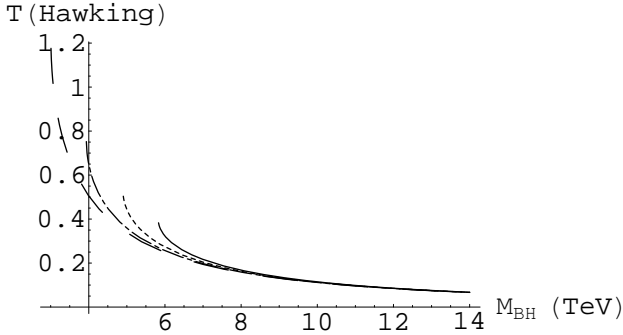


FIG. 17: Range of Hawking temperature, in TeV, produced in LHC by a charged spinning BH evaporation versus BH mass interval. The range covers some values of charge for a fixed angular momentum: full line: $Q^* = -2M$, dotted line: $Q^* = -1.5M$, dashed-dotted: $Q^* = -M$, dashed: $Q^* = -0.5M$, all for spinning parameter $a = 0.5$. This means the BH charge is an important parameter in the determination of exact BH decay. Here, the Planck mass is 4 TeV and $n = 6$ for Randall-Sundrum braneworld model.

In this case, as the charge Q^* increases, the Hawking temperature increases as well. On the other hand, as the mini-BH mass increases, T_H decreases, and for all values of Q^* , Hawking temperature T_H tends to the same value as M_{BH} is ~ 10 TeV.

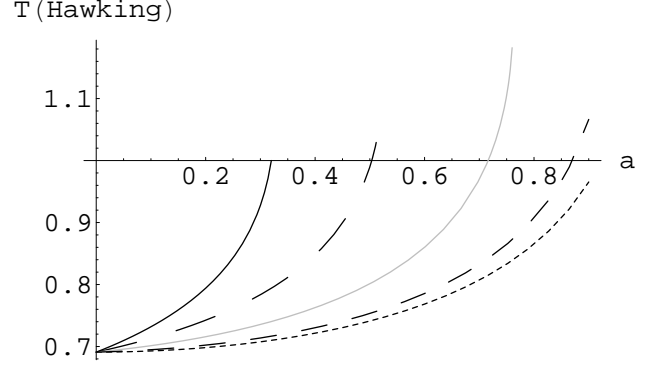


FIG. 18: Range of Hawking radiation, in TeV, produced in LHC by a charged spinning BH evaporation versus BH spinning parameter a , for a fixed BH mass (1 TeV) for Randall-Sundrum braneworld model. Here we present an example for $M_P = 4$ TeV. For the very short dashed line: $Q^* = -0.1$; for the short dashed line: $Q^* = -0.5$; for the full gray $Q^* = -1$; for the full black line: $Q^* = -1.5$; and for the long dashed line: $Q^* = -2$. In this case, the spinning of the BH is constrained by the electric and tidal charge.

It is notorious by the graphic above that as the spin parameter a increases T_H increases as well.

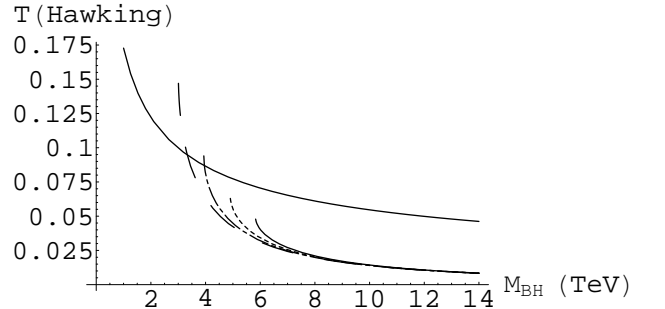


FIG. 19: Range of Hawking radiation, in TeV, produced in LHC by a charged spinning BH evaporation versus BH spinning parameter a , for a fixed BH mass (1 TeV) for Randall-Sundrum braneworld model, where there is one extra warped dimension. Here we present an example for Planck mass $M_P = 1$ TeV. For the very short dashed line: $Q^* = -0.1$; for the short dashed line: $Q^* = -0.5$; for the full gray $Q^* = -1$; for the full black line: $Q^* = -1.5$; and for the long dashed line: $Q^* = -2$. In this case, the spinning of the BH is constrained by the electric and tidal charge.

In this case Hawking temperature is about $\sim 10^1 - 10^2$ GeV. The superior line indicates the graphic for a Schwarzschild BH Myers-Perry model [25, 26].

IV. CONCLUDING REMARKS AND OUTLOOKS

By solving Einstein equations on the brane we have found an explicit form for the perturbation of a Kerr-Newman metric singularity, corresponding to a Kerr-Newman BH. Such perturbation gives rise to more realistic calculations and predictions involving mini-BH production at LHC, since we concern braneworld corrections as they would appear in the measurement of cross sections, Hawking temperature and some other physical observations related to the mini-BH production at LHC. Mini-BHs are a natural choice for testing braneworld formalisms: the lighter the BH the greater its surface gravity is, and consequently mini-BHs are more sensitive on braneworld physical effects, like, e.g. Hawking evaporation in the context of Randall-Sundrum and ADD models. Also, the 14 TeV maximum energy of LHC could allow it to become a mini-BH factory with a production rate as high as about one per second, and after formation, mini-BHs are expected to lose the information associated with multipole and angular momenta, to decay via Hawking radiation, and eventually either disappear completely or leave Planck-sized remnant [25, 52, 53, 54] around Planck masses $M_P = 1.6$ TeV for $D = 6$ and $M_P = 0.25$ TeV for $D = 10$ [55].

Figures (1), (2), and (3) in Section II show the corrections of Schwarzschild radius as functions of the spin parameter a , for different values of the charge Q^* . Such brane effect-corrections arise immediately from the solutions of Einstein equations on the brane and they can describe more realistic predictions concerning mini-BHs features at LHC.

Brane effect-corrections for mini-BH cross sections at LHC are shown in Figure (4) for $n = 3$ extra dimensions in ADD model. Figures (5), (6), (7), (8), and (9) also show the increment of the brane effect-corrected Kerr-Newman BH horizons and the respective cross section as functions of the Kerr-Newman BHs, for different values of charge, as functions of BH and Planck masses, and spinning parameter, respectively for the Randall-Sundrum model and the $n = 2$, $n = 4$, and $n = 6$ extra-dimensional ADD models. In all these graphics the wide gray line indicates the standard Myers-Perry approach for Schwarzschild BHs, and it is clear that our results coming from Kerr-Newman analysis predict more accurately the results obtained from the static Schwarzschild Myers-Perry approach.

Figures (10), (11), (12) definitively express the brane corrections in mini-BH cross sections as functions of BH mass and the effective Planck mass in the range 1-10 TeV of LHC, respectively for Schwarzschild — in Myers-Perry model — and Kerr-Newman mini-BHs for various values of extra dimensions both in Randall-Sundrum and ADD models. Figures (13), (14), and (15) show the brane corrections in mini-BH cross sections as functions of the spinning parameter a and the mini-BH mass, in the range 1-10 TeV of LHC, respectively for various values of n ex-

tra dimensions in Randall-Sundrum ($n = 1$), and ADD models (the particular cases where $n = 2$ and $n = 6$). Our results show prominent effects of higher number of extra dimensions in cross sections and Hawking temperature associated with Kerr-Newman mini-BHs. Fig.(16) illustrate the brane-effect corrected cross sections as functions of Q^* and the spinning parameter a , showing how the charge constrains the mini-BH spin.

Figures (17), (18), and (19) illustrate Hawking temperature of a Kerr-Newman mini-BH respectively as function of BH mass and spinning parameter a , both in an extra-dimensional scenario. Figure (18) show the braneworld Kerr-Newman mini-BH corrections of Schwarzschild Myers-Perry model. Again, for different values of mini-BH charge, T_H seems not to be sensitive to these values for higher values of mini-BH mass, above 10 TeV. This shows that in the range 1-10 TeV of LHC resides our brane effect-corrections. Also, from Figs. (17), (18), and (19), as the charge Q^* increases, the Hawking temperature increases as well. On the other hand, the greater the mini-BH mass, the lower the T_H , and for all values of Q^* , Hawking temperature T_H tends to the same value as M_{BH} is ~ 10 TeV. This allows us to follow [26, 27] in a braneworld viewpoint, and to predict — from the graphic of Hawking temperature $\times M_{BH}$ — the Planck mass and the number n of extra dimensions in a more precise and realistic approach. The fact that T_H increases as the spinning parameter a also increases shows that all more realistic brane effect-corrections obtained in this paper for cross sections, Hawking temperature, and Kerr-Newman horizons, can be extremely useful in an optimal use of the range 1-10 TeV of LHC.

The question concerning where mini-BHs mostly radiate — on the brane or on the bulk — remains open, and its answer will allow, at least in principle, to detect most of the decay products in the case of radiation on the brane. If BHs radiate mostly in the bulk, one has to take any constraints coming from non-observation of such events with caution. Due to non-zero impact parameter of initial particles, most of the produced BHs will be highly rotating, and our model described in this paper describe such processes with more accuracy. Such BHs radiate mostly in the bulk — at least as long as they rotate fast — as first pointed out in [35, 36, 37]. The reason is that rotating BHs exhibit the effect of superradiance, i.e., certain radiation modes get significantly amplified taking away rotational energy of the BH [58, 59, 60, 61]. The model described here using Kerr-Newman electrically charged and rotating BHs gives rise to a more profound analysis of superradiance effect, which is highly spin-dependent and radiation of higher spin particles is highly preferred — for instance gravitons are preferred to photons and neutrinos. While on a brane with (1+3)-dimensions there are two degrees of freedom of graviton, the probability of a graviton emission is a hundred times higher than the probability of a photon and neutrino emission [56, 57]. This effect must be even stronger in the higher-dimensional case where you have more gravi-

itational degrees of freedom available. Also, superradiance exists in higher dimensions [60, 61] and interactions between BHs and the brane [62, 63, 64, 65] exist, in which BH lose some of its components of angular momentum. Thus, after initial bulk domination, in final stages, it is expected again that brane radiation dominates. Also, the extra-dimensional geometrical scenario is fundamental for such analysis. New analysis procedures involving superradiance and charged Kerr-Newman mini-BHs are to be discussed in a forthcoming paper.

Besides, gravitational field equations on the brane can also be solved in the framework of ADM [67] formalism. For instance, in [68] SMS effective gravitational field equation on the brane was recovered and it generalized the off-brane equations due to both the acceleration of the timelike 1-form fields, in the presence of general bulk energy-momentum tensor.

Finally, a mini-BH demands a sufficiently large entropy S_{BH} [66], where the threshold mini-BH mass is much larger, and therefore the production becomes much smaller at the LHC. For example, to require $S_{BH} > 25$

implies $M_{BH} > 5M_P$ and if one assumes $M_P = 2$ TeV, then the threshold mini-BH mass is about 10 TeV, again around the limit energy in LHC.

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